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Trend prey predator model - Analysis of gause model

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A B S T R A C T

Any application of PP (Prey - Predator) models based on nonlinear differential equations requires identification of numerical values of all constants. This is often a problem because of severe information shortages. Many PP models are numerically sensitive and/or chaotic. Moreover, complex PP tasks are based on integration of differential equations with (partially) unknown numerical values of relevant constants and vague heuristics, e.g. vaguely described capture rate. These are the main reasons why PP numerical simulations cannot identify all important/relevant features, e.g. attractors. Trend models use just three values namely *positive (increasing), zero (constant), negative (decreasing).* A multiplication of a trend variable X by a positive constant *a* is irrelevant, it means that $aX = (+)X = X$. This obvious equation is used to eliminate all positive multiplicative constants *a* from PP mathematical models. A solution of a trend model is represented by a set of scenarios and a set of time transitions among these scenarios. A trend analogy of a quantitative phase portrait is represented by a discrete and finite set of scenarios and transitions. A trend version of the well-known Gause PP model is studied in details. The provably complete set of 41 scenarios and 168 time transitions among them are given.

1. Introduction

Most of mathematical models describing physical or natural processes are formulated in terms of nonlinear differential equations, see e.g. Ibragimov and Ibragimov (2009). Prey predator studies are important tools to solve a broad spectrum of different task – biology, economics, ecology, see e.g. Focardi and Rizzotto (1999). An important task of mathematical biology is to model complex biological systems (Devloo et al., 2003). Unfortunately, if just one component is ignored then the results can be totally misleading. This is the key reason why PP models have received considerable attention in scientific literature, see e. g. (Krivan and Priyadarshi, 2015; Zhang and Shen, 2015).

A set of ordinary and/or partial nonlinear differential equations are frequently used descriptions of unsteady state behaviours of many different PP systems e. g. (Martín-Fernández et al., 2014; Vanegas-Acosta and Garzon-Alvarado, 2014). Numerical evaluations of ONDEs (ordinary nonlinear differential equations) constants are very often an information-intensive task, see e.g. (Douc et al., 2014; Ljung, 1999). It means that extensive data sets are required. However, PP systems are often poorly known.

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A paradigmatic three-dimensional Lorenz model has been frequently studied. It is a well-known fact that different sets of numerical values of its constants determine its chaotic/non-chaotic behaviours. Therefore, a study of different sets of constants is inevitable, see e.g. (Yang et al., 2014).

Numbers are infinitely precise. This precision is a substantial disadvantage if systematic studies of multidimensional chaotic behaviours must be performed. Extreme sensitivity of some chaotic models makes it very difficult to characterize completely all relevant features, e.g. attractors, strange attractors, repellers of chaotic behaviours of systems under study.

Model *M* has *n* variables. Each variable is quantified by *s* different values. For example the following verbal quantifiers are used, $s = 4$:

Very low, low, medium, high

It means that there are λ

$$\lambda = s^n \tag{1}$$

different values.

2. Trend models

The concept of qualitative analysis was introduced into artificial intelligence approximately 30 years ago, see e.g. Kuipers (1994); The used quantifiers are not numbers but trends – increasing, constant, decreasing. However, Qualitative analysis is frequently used for study of models, data sets etc. based on traditional numerical quantifications, see e.g. (Ji et al., 2011; Zu, 2013). A possible elimination of different miss-understandings a term trend analysis is introduced to make sure that models based on the trend values are clearly distinguished from conventional numerically based models.

Naïve physics generated several algorithms, for additional details see (Dohnal, 1991; Kuipers, 1994; Salles et al., 2006; Suc et al., 2004). However, the original interpretation did not attract attention of broad spectrum of potential users. The key reason is a complicated integration of equation-based and equation-less knowledge items. E.g. a vague heuristic is not described by an equation – *If a number of wolfs is increasing then a number of rabbits is decreasing* However, such vague equation-less heuristic cannot be simply ignored, it must be incorporated into a trend model; the information shortage is critical

The algorithm presented in this paper eliminates this important problem.

Analysis presented in this paper is based on three trend values, i.e. s = 3, see (1):

$$\begin{array}{cccc} \text{Values:} & \text{Positive} & \text{Zero} & \text{Negative} \\ \text{Derivatives:} & \text{Increasing} & \text{Constant} & \text{Decreasing} \\ & + & 0 & - \end{array} \tag{2}$$

A set of n numerical variables

$$x_1, x_2, \ldots x_n \tag{3}$$

is replaced by the corresponding set of trend variables:

$$X_1, X_2, \ldots X_n \tag{4}$$

A trend solution of ONDEs is specified if all its *n* trend variables are described by the corresponding trend triplets:

$$(X_1, DX_1, DDX_1), (X_2, DX_2, DDX_2), \ldots (X_n, DX_n, DDX_n), \tag{5}$$

where X_i is the *i*th variable and DX_i and DDX_i are the first trend and second trend derivations with respect to time *t*.

A simple transfer of quantitative derivative dx_i/dt to the trend derivatives DX is:

$$\begin{array}{l} \text{if } dx_i/dt > 0 \text{ then } DX_i = + \\ \text{if } dx_i/dt = 0 \text{ then } DX_i = 0 \\ \text{if } dx_i/dt < 0 \text{ then } DX_i = - \end{array} \tag{6}$$

However, PP models have often their variables positive because of their very nature. E.g. a population of a predator is always positive. It means that the following scenarios are studied:

$$(+, DX_1, DDX_1), (+, DX_2, DDX_2), \ldots (+, DX_n, DDX_n) \tag{7}$$

The third DDDX and higher derivatives are ignored (5, 7).

There are *N* distinguishable trend scenarios if the third derivatives and higher are ignored (5):

$$N = 3^{(3)n} \tag{8}$$

where n is the dimensionality, see (5, 7).

It is possible to include the third and higher derivatives into trend modelling to increase the discriminative power of the results. However, the total number of scenarios N will be very high and it would be difficult to interpret the results. More importantly PP processes are often ill-known and the third derivatives are excessively inaccurate. However, the below given analysis can formally incorporate derivatives of any order.

The following set of quantitative differential equation represents the well-known and extensively studied chaotic Lorenz model, see e.g. Bai and Lonngren (2000):

$$\begin{aligned} \frac{dx}{dt} &= -ax + ay, \\ \frac{dy}{dt} &= rx - y - xz, \\ \frac{dz}{dt} &= -bz + xy, \end{aligned} \tag{9}$$

Any positive multiplicative constant A satisfies the following relation:

$$AX = (+)X = X \tag{10}$$

The trend interpretation of the equation (9) is therefore, see (10):

$$\begin{aligned} DX &= -X + Y \\ DY &= X - Y - XZ \\ DZ &= -Z + XY \end{aligned} \tag{11}$$

The classical set of Lorenz equation (9) is trend represented by the set (11). However, the same trend model (11) represents the following quantitative generalized Lorenz models:

$$\begin{aligned} dx/dt &= -c_1x + c_2y \\ dy/dt &= c_3x - c_4y - c_5xz \\ dz/dt &= -c_6z + c_7xy \end{aligned} \tag{12}$$

where c are any numerical positive constants

Let set of m trend n-dimensional scenarios (5):

$$S(m, n) \tag{13}$$

be a solution of a trend n dimensional model M, see (4):

$$M(\mathbf{X}, n) \tag{14}$$

Any set of scenarios (13) is a finite discrete set. The reason is the discrete nature of the quantifiers (2).

The list of scenarios is complete and therefore no unsteady state behaviour of a PP system is overlooked if the analysis is based on a relevant trend model (14), e.g. (11). An example is the trend steady state scenario based on the first and second derivatives

$$(+00)_1; (+00)_2 \ldots (+00)_n, \text{ see } (2, 7). \tag{15}$$

The following example represents a three dimensional ($n = 3$) set of two ($m = 2$) scenarios (5, 7):

$$\begin{array}{l} X_1\ X_2\ X_3 \\ 1\ (+++)\ (++0)\ (+--) \\ 2\ (+++)\ (++-)\ (+--) \end{array} \tag{16}$$

A common sense interpretation of the first scenario (16) is:

Variable X_1 is increasing more and more rapidly and variable X_2 is increasing linearly and variable X_3 is decreasing more and more rapidly.

Algorithms of trend reasoning are based on a complex mathematics of combinatorial nature and/or different algorithms related to logic, see e.g. (Mueller, 2015). Specific computer programs are available to solve trend models, see e.g. Garp3 (Bredeweg et al., 2009; Salles and Bredeweg, 2009). It is not the goal of this paper to study this problem.

However, a trivial algorithm based on screening all possible scenarios can be presented. There are N (8) distinguishable scenarios. Each scenario is tested. If the scenario is a solution of the model (14) then it is assigned to the set of scenarios S (13).

A trivial example of trend mathematics is a trend solution of the following equation:

$$Y + Z = X$$

It is clear that if $Y = +$ and $Z = +$ then X must be positive.

2.1. Trend arithmetic operations

The following description of Trend Algebra is sufficient to make this paper self-contained. For additional details see (Dohnal, 1991; Kuipers, 1994; Salles et al., 2006; Suc et al., 2004).

A trend addition

$$X_i + X_j = X_s \tag{17}$$

is represented by the following matrix (see Table 1):

It is sometimes possible to find more than one trend value. It is impossible to predict a sign of the result:

$$(X_i = positive) + (X_j = negative) = (X_s = ?), \text{ see } (12) \tag{18}$$

A trend derivative of a sum of trend variables is a sum of their trend derivatives.

$$\begin{aligned} DX_i + DX_j &= DX_s \\ DDX_i + DDX_j &= DDX_s \end{aligned} \tag{19}$$

A trend multiplication

$$X_i * X_j = X_s \tag{20}$$

is described by the following table (Table 2):

A known relation for the first trend derivation gives

$$D(X_i * X_j) = X_i * DX_i + X_j * DX_j = DX_s \tag{21}$$

The second trend derivative of the product (2) is the first derivative of the first derivative

$$D(DX))$$

2.2. Trend transitions

Fig. 1 gives a trend description of an oscillation using the one dimensional triplets $n = 1$ (2, 5).

A complete set of all possible one-dimensional transitions is given in the following table:

E. g. the third line of Table 1 indicates that it is possible to transfer the triplet (++ -) into the triplet (++ -). This transition is not the only possible. There are two more, namely (+0 -), (+0 0).

There are several one-dimensional transitions presented in the graphical form, see Fig. 1. For example the following transition (+0 -) → (+- -) represents the transition from the peak, see Fig. 1. All these transitions correspond to the Table 3.

A transitional graph G is an oriented graph. Its nodes are the set of scenarios S (13) and oriented arcs are the transitions T:

$$G(S, T(S)) \tag{22}$$

A PP oscillation can be represented by a simple transitional graph G (22), see Figs. 1 and 2. The transition from the triplet (++ +) to the triplet (++ 0), see Fig. 2, corresponds to the first row of Table 1, see the transition 1a.

The transitional graph given in Fig. 2 is a time sequence of one-dimensional scenarios. The graph is based on trend values. It means that it is not known how long it takes to move from one scenario to another. However, the time sequence is known

Table 1
Trend Addition. Source: Trivial arithmetics.

+	X_j			
X_i		+	0	−
	+	+	+	?
	0	+	0	−
	−	?	−	−

Table 2
Trend Multiplication. Source: Trivial arithmetics.

*	X_j			
X_i		+	0	−
	+	+	0	−
	0	0	0	0
	−	−	0	+

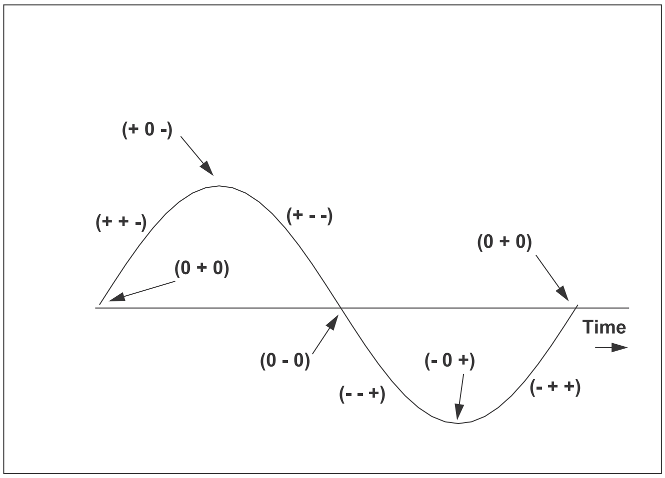

Fig. 1. A trend description of an oscillation.

and uniquely described. For example it is not possible to move from the scenario (++ +) to the scenario (++ -) by-passing the scenario (++ 0), see Fig. 2.

Table 3 is not a dogma. The only requirement is that the transitions given in Table 3 must be accepted by a user. The set of possible one-dimensional transitions is a suggestion. A potential rectification of the carefully study of the resulting transitional graph (22). If some transitions are (partially) unrealistic then the Table 3 must be rectified for this specific case.

Table 3
All possible one-dimensional transitions.

	From		To	Or	Or	Or	Or	Or	Or
1	+ + +	→	+ + 0						
2	+ + 0	→	+ + +	+ + -					
3	+ + -	→	+ + 0	+ 0 -	+ 0 0				
4	+ 0 +	→	+ + +						
5	+ 0 0	→	+ + +	+ - -					
6	+ 0 -	→	+ - -						
7	+ - +	→	+ - 0	+ 0 +	+ 0 0	0 - +	0 0 +	0 0 0	0–0
8	+ - 0	→	+ - +	+ - -	0–0				
9	+ - -	→	+ - 0	0 - -	0–0				
10	0 + +	→	+ + 0	+ + -	+ + +				
11	0 + 0	→	+ + 0	+ + -	+ + +				
12	0 + -	→	+ + -						
13	0 0 +	→	+ + +						
14	0 0 0	→	+ + +	- - -					
15	0 0 -	→	- - -						
16	0 - +	→	- - +						
17	0–0	→	- - 0	- - +	- - -				
18	0 - -	→	- - 0	- - +	- - -				
19	- + +	→	- + 0	0 + +	0 + 0				
20	- + 0	→	- + -	- + +	0 + 0				
21	- + -	→	- + 0	- 0 -	- 0 0	0 + -	0 0 -	0 0 0	0 + 0
22	- 0 +	→	- + +						
23	- 0 0	→	- + +	- - -					
24	- 0 -	→	- - -						
25	- - +	→	- - 0	- 0 +	- 0 0				
26	- - 0	→	- - -	- - +					
27	- - -	→	- - 0						

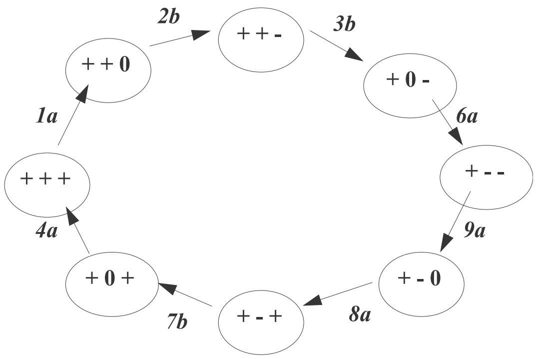

Fig. 2. Transitional graph — oscillation and transitions given in Table 1.

3. Gause trend model

Gause PP model is, see e.g. (Gonzales-Olivares and Rojas-Palma, 2011; Kooij and Zegeling, 1996):

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - p(x)y, \\ \frac{dy}{dt} &= y[-\delta + \gamma p(x)]. \end{aligned} \tag{23}$$

where p(x) is the capture rate.

Let us suppose that the capture rate p(x) is not known in a form of an equation. Common sense reasoning, is used to specify its equationless trend interpretation $P(X)$:

- The first derivative of P(X) with respect to X is positive; P(X) is increasing;
- The second derivative of P(X) with respect to X is positive; P(X) is increasing more and more rapidly (24)
- P(X) is not negative
- There is a positive delay

The graphical representation of the trend capture rate P(X) is given in Fig. 3:

All three numerical constants α, δ and γ (23) are positive.

Trend interpretation of Eq. (23) is, see (10):

$$\begin{aligned} \mathrm{DX} &= \mathrm{X} - \mathrm{P(x)Y} \\ \mathrm{DY} &= -\mathrm{Y} + \mathrm{P(x)} \end{aligned} \tag{25}$$

The numbers of rabbits/prey X and foxes/predators Y are always positive. Therefore the corresponding triplets have the first value always positive, see (2, 7):

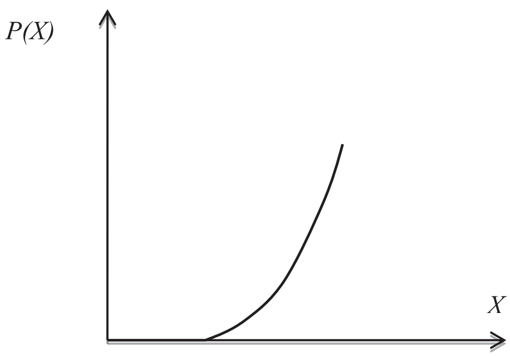

Fig. 3. Representation of the trend capture rate P(x).

$$(+, ?, ?) \tag{26}$$

where the question mark indicates that the first and the second derivatives must be evaluated. Each question mark (26) can have three trend values (2). It means that there are 9 different trend scenarios (26). The model (25) has two variables X and Y; therefore, two triplets (26) are required. It gives N = 9 x 9 = 81 distinguishable scenarios (8):

$$\begin{array}{cc} X & Y \\ (+\ ??) & (+\ ?\ ?) \end{array} \tag{27}$$

It means that if no model is given, i.e. no restrictions, then there are 81 scenarios

$$\begin{array}{lll} & X & Y \\ 1 & (+\ +\ +) & (+\ +\ +) \\ 2 & (+\ +\ +) & (+\ +0) \\ 3 & (+\ +\ +) & (+\ +\ -) \\ 4 & (+\ +0) & (+\ +\ +) \\ \ldots\ldots\ldots \\ \ldots\ldots\ldots \\ 81 & (+\ -\ -) & (+\ -\ -) \end{array}$$

Table 4 gives $m = 41$ (13) scenarios of the trend Gause model (25). The Gause model (25) eliminates 81–41 = 40 distinguishable scenarios. It means that nearly 50 per cent of the distinguishable scenarios (27) is eliminated.

Table 4 summarises all possible Gause behaviours. For example:

- *there is the steady state scenario (16), zero derivatives, see No. 21*
- *variable X is increasing more and more rapidly and so does Y, see scenario No. 1*
- *Variable X is increasing more and more rapidly and variable Y is decreasing more and more rapidly, see No. 7.*

There are 168 transitions T (22) among 41 scenarios, see Table 4. As an example a set of 21 transitions among the scenarios, see Table 4, is given in Table 5.

The first row of Table 5 indicates that the transition from the first scenario, see Table 4, to the second scenario is possible. It is clear that the variable X is not changed during this transition, see Table 4. The variable Y is changed:

$$\begin{array}{ccc} Y & & Y \\ (+\ +\ +) & \rightarrow & (+\ +0) \end{array} \tag{28}$$

A sub-graph of the Gause transitional graph G is given in Fig. 4.

A time sequence of the following scenarios, see Table 4:

$$21 \rightarrow 1 \rightarrow 2 \rightarrow 8 \rightarrow 9 \rightarrow 15 \rightarrow 21 \tag{29}$$

Table 4
List of 41 scenarios of Gause PP model.

No	X	Y	No	X	Y
1	+ + +	+ + +	22	+ 0 -	+ + +
2	+ + +	+ + 0	23	+ 0 -	+ + 0
3	+ + +	+ + -	24	+ 0 -	+ + -
4	+ + +	+ 0 +	25	+ - +	+ + -
5	+ + +	+ - +	26	+ - +	+ 0 -
6	+ + +	+ - 0	27	+ - +	+ - +
7	+ + +	+ - -	28	+ - +	+ - 0
8	+ + 0	+ + +	29	+ - +	+ - -
9	+ + 0	+ + 0	30	+ - 0	+ + -
10	+ + 0	+ + -	31	+ - 0	+ 0 -
11	+ + 0	+ 0 +	32	+ - 0	+ - +
12	+ + 0	+ - +	33	+ - 0	+ - 0
13	+ + -	+ + +	34	+ - 0	+ - -
14	+ + -	+ + 0	35	+ - -	+ + +
15	+ + -	+ + -	36	+ - -	+ + 0
16	+ + -	+ 0 +	37	+ - -	+ + -
17	+ + -	+ - +	38	+ - -	+ 0 -
18	+ 0 +	+ - +	39	+ - -	+ - +
19	+ 0 +	+ - 0	40	+ - -	+ - 0
20	+ 0 +	+ - -	41	+ - -	+ - -
21	+ 0 0	+ 0 0			

Table 5
First 21 transitions of Gause model. Source: Own Compilation.

No	From, Table 4	To, Table 4
1	1	2
2	1	8
3	1	9
4	2	1
5	2	3
6	2	8
7	2	9
8	2	10
9	3	2
10	3	9
11	3	10
12	4	1
13	4	8
14	5	4
15	5	6
16	5	11
17	5	12
18	6	5
19	6	7
20	6	12
21	7	6

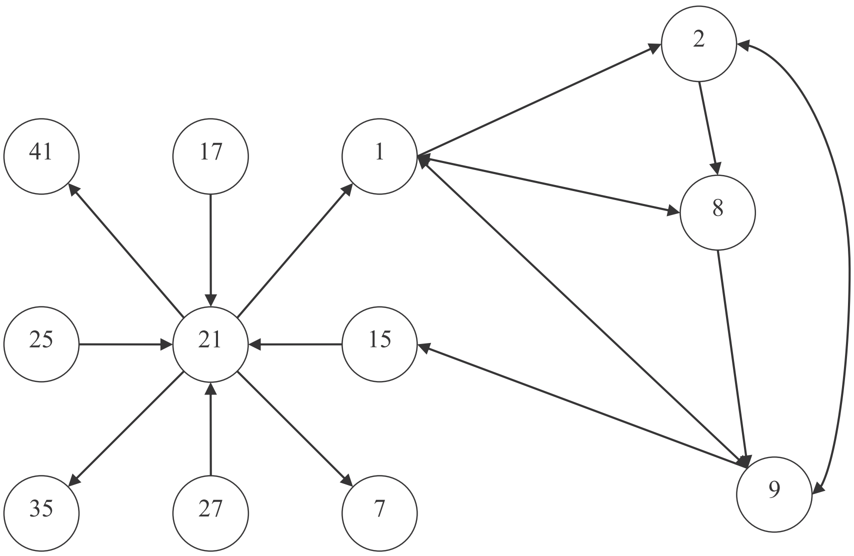

Fig. 4. Example of a loop. Source: Own compilation.

represents e.g. a trend loop. The loop (29) represents a sort of a Gause oscillation, see Figs. 1 and 2. The complete transitional graph represents all possible time sequences of Gause scenarios.

4. Conclusions

Many important PP problems described by ONDEs are ill-known and/or very sensitive. It is therefore often impossible to use traditional numerical mathematics, to solve PP ONDEs.

Trend models are applicable under conditions of severe information shortages as they are based on trends only. However, trend models can give just trend solutions. For example the following queries can be answered:

- Is it possible to increase both PP populations X and Y at the same time?
- If the current situation is — *both PP populations are decreasing* — is it possible to reach the scenario - prey is increasing predator is decreasing?
  - o Are there several different trend paths?
  - o Which variables must be changed?

The trend Gause model can be used to reject or accept some NTSs (numerical time series) of measurements as an element of Gause unsteady state behaviour. Conventional filtration methods are used to transfer NTSs into FNTS (Filtered NTS), see e.g. (Douc et al., 2014; Loire and Mezić, 2013). FNTS can be transformed into a sequence of trend scenarios TTS (trend time series). If the graph given in Fig. 1 would be a FNTS then TTS is given in Fig. 2.

If the TTS is not a subsequence of the Gause transitional graph then the corresponding observation is not a Gause trend feature.

Gause model was chosen as a case study. However, there are many other PP models. Therefore similar studies, as the one given above, can be performed. Twenty published PP models are being trendly analysed. It means that intersection of all pairs of sets of scenarios can be identified. The pairwise intersections can be used to quantify trend similarities of two PP model.

Declarations of interest

None.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.gecco.2019.e00634.

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